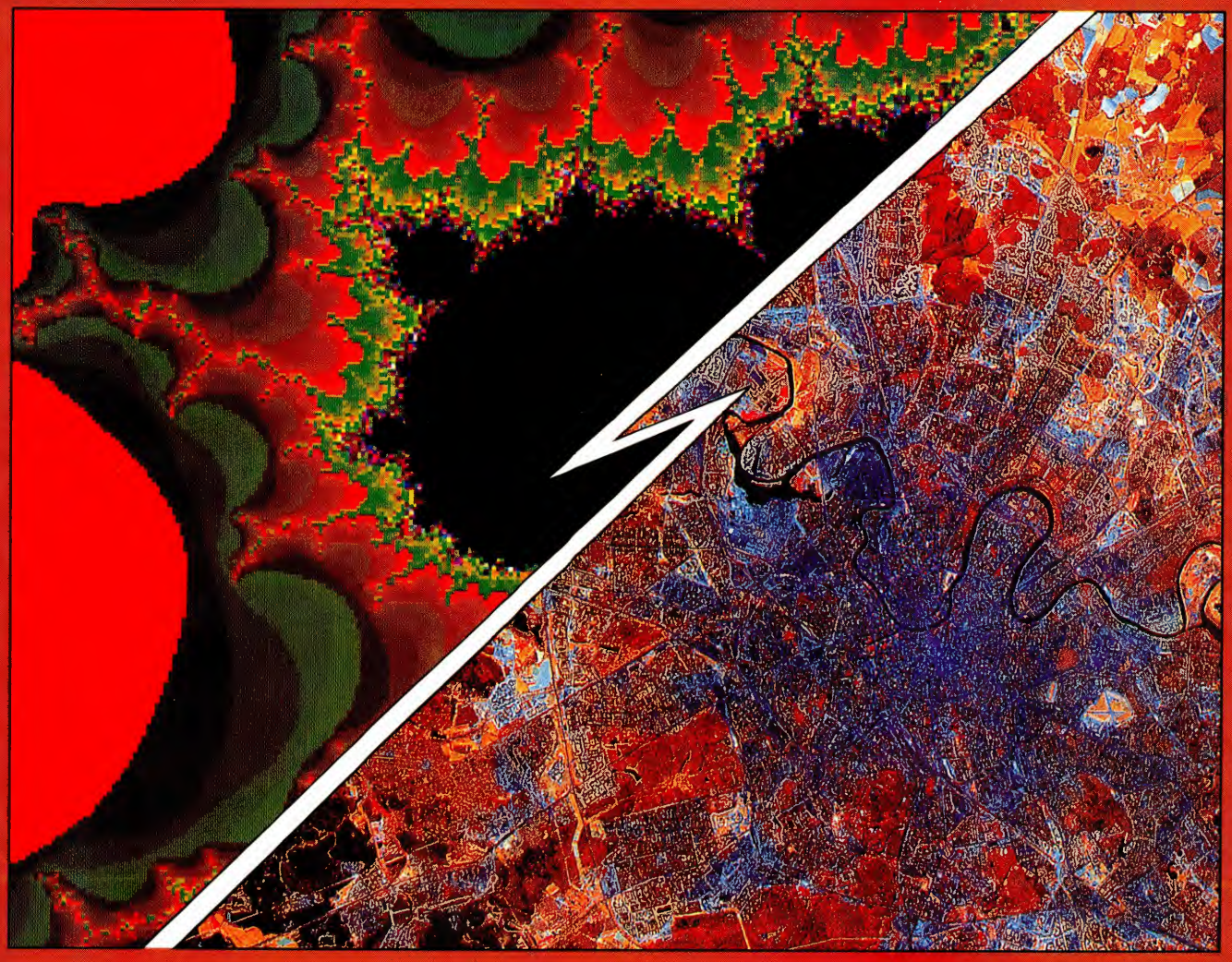

ORDER TO...

...RANDOMNESS

CenturyMATHS

developed at the Institute of Education,
University of London

ORDER TO . . . CHAOS
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PATTERNS IN . . . RANDOMNESS
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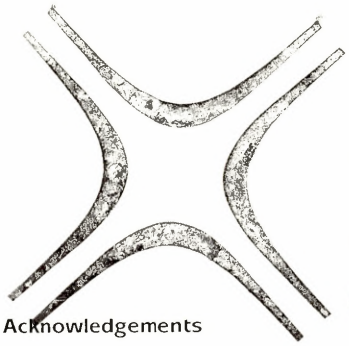

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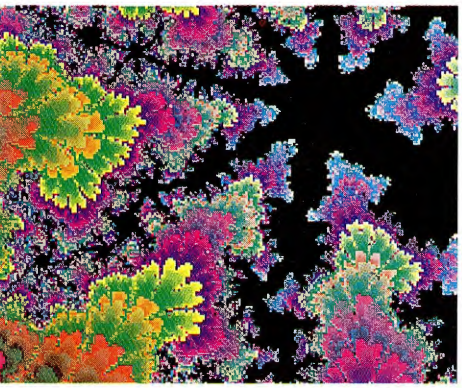
ORDER TO ... CHAOS

The first four sections will provide the ideas to understand the final section 'Towards chaos'

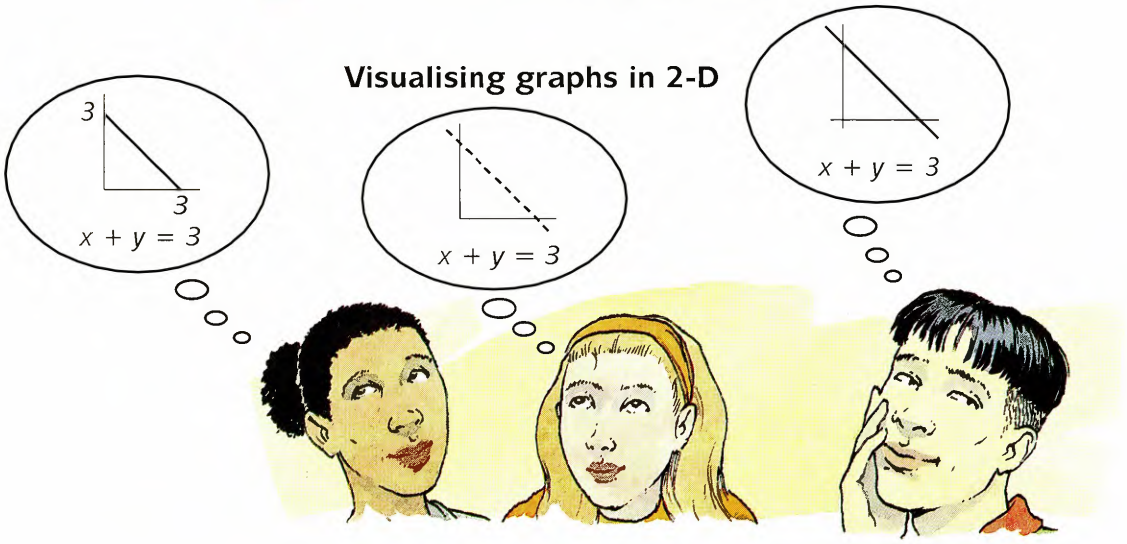


Symbols in the margin

- Special things you *will* need
- Special things you *may* need
- More ideas
- Link page
- Worksheet reference
- Work for graphic calculator here
- Work for computer here (screen may show one of these: D Database G Graph plotter S Spreadsheet L Logo)
- LD Logo 2000 disk
- LN Logo 2000 documentation
- LP LogoPack

Visualising graphs

Visualising graphs in 2-D



Work in a small group for this activity.

You are going to try to imagine what a graph looks like without drawing it.

- Imagine the graph of $y = x$

  ... then imagine the graph of $y = x + 4$ drawn on the same diagram.

  Finally, imagine the graph of $x + y = 10$.

  Describe the graphs to each other.

  Discuss what the graphs look like until you agree!

- This time, imagine the graph of $y = 4x$

  ... then imagine the graph of $y = \frac{1}{4}x$

  ... and finally imagine the graph of $y = x$.

  Describe these graphs to each other.

- Discuss any similarities or differences in both sets of graphs.

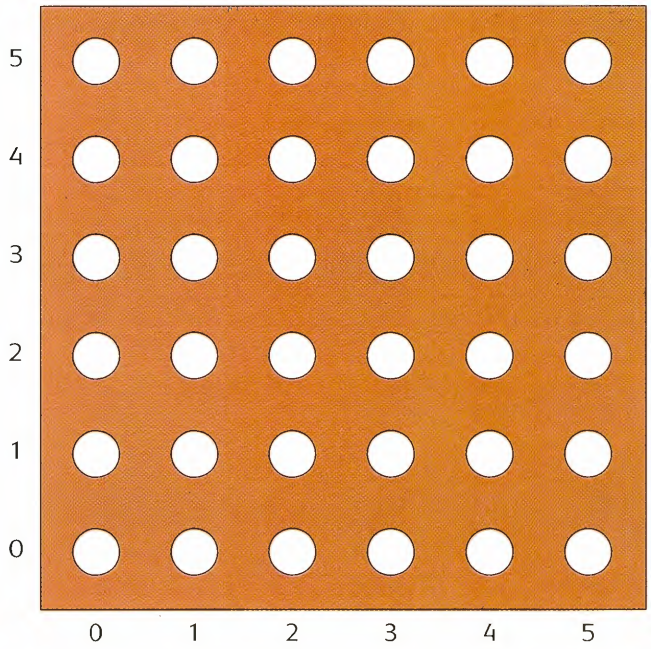
- Now try something harder – try thinking about three dimensions.

Graphs in 3-D

Multilink

base board/pegboard

The holes of a pegboard are numbered from 0–5.



For the point (2, 3) a column of blocks 2 + 3 high is placed on the hole.

For each hole add the coordinates together and build a column on the hole.

Make a model, filling a 5 × 5 grid.

- Look at your model from the side. What can you see?

Convince each other.

straws

Blu-tack

squares of card 4 cm × 4 cm

Practical points

Effective models can also be made using straws.

Make a base by pressing a thin square of plasticine or Blu-tack onto the square of card.

Mark the base with a square grid.

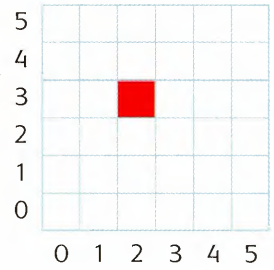
For the square (2, 3) cut a straw of length 5 cm and press into the base.

page 22

Graphs in colour

multilink

You can use colour as a third dimension.



Key
0 Black
1 Green
2 Blue
3 Yellow
4 Orange
5 Red

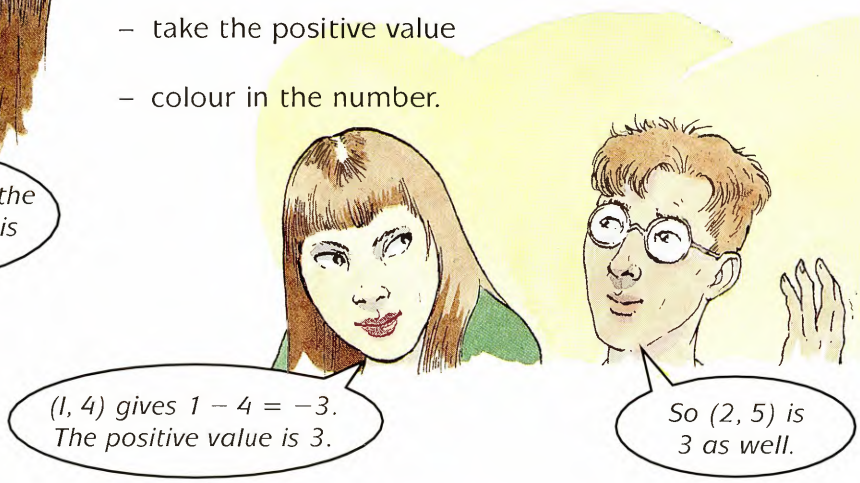
- On squared paper, label a grid as shown above.

Instead of using multilink cubes, use a colour to represent each number on your graph.

Complete a colour graph on your grid.

Now try these either using multilink or by colouring squares.

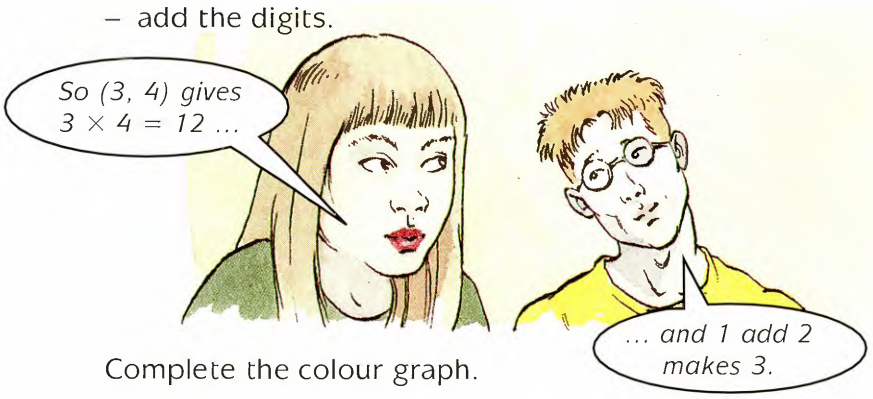
- For the coordinates of each square (x, y) on a grid:
  - work out $x - y$
  - take the positive value
  - colour in the number.



This is called the **modulus** written $|x - y|$.

- Try to describe the graph to your partner before you make or colour it.

- For each square (x, y) on a grid:
  - work out xy,
  - add the digits.



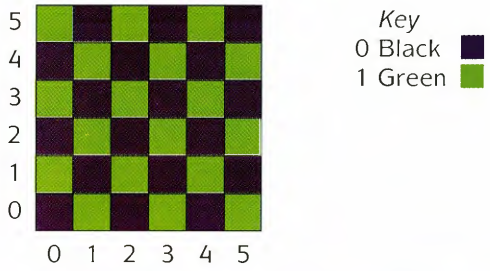
Complete the colour graph.

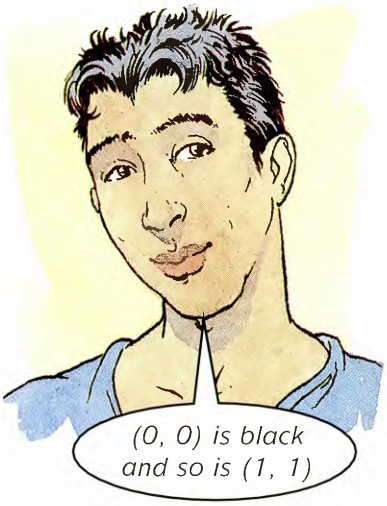
Teaser time

What do you do to the coordinates to get these graphs?

1

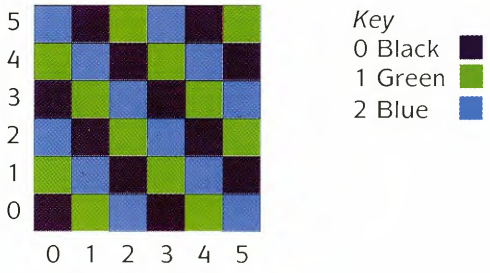
- Rule 1:





Invent a rule that would give exactly this graph.

- Rule 2: This rule makes this colour graph.



Invent a possible rule.

1

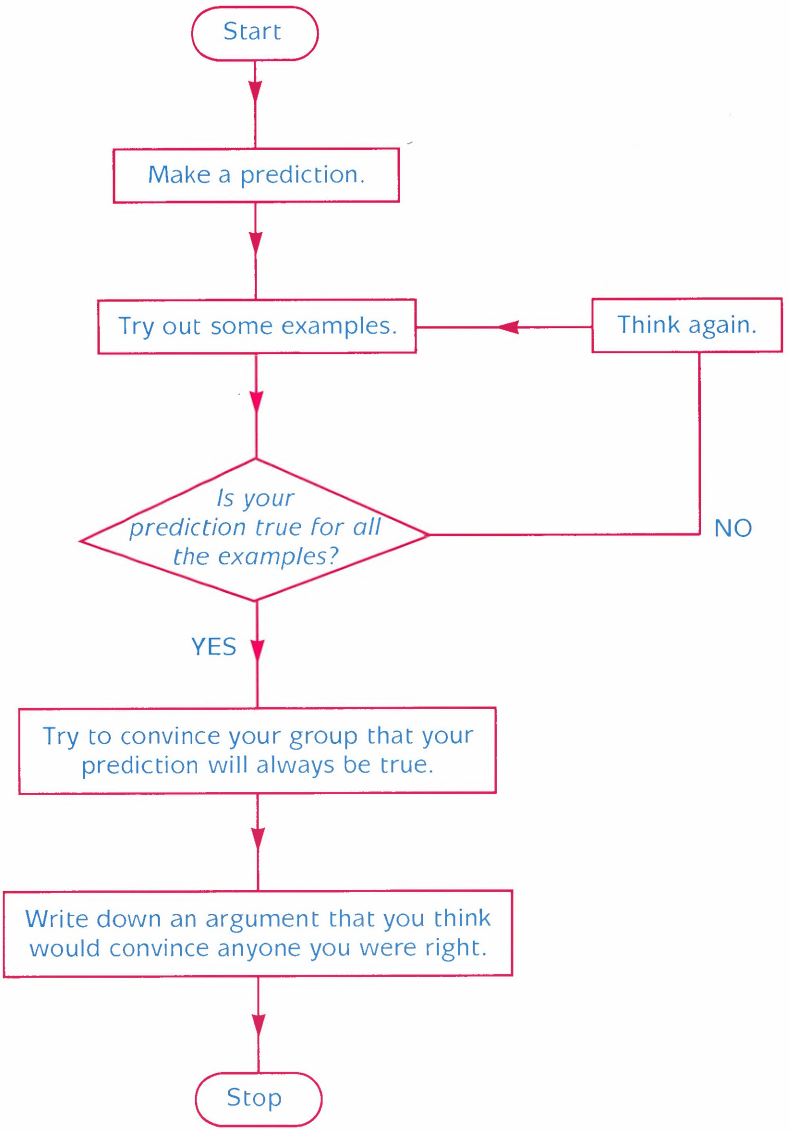
For a further idea, look at the Worksheet 1.

. . . Graphs in colour

Prediction

A **prediction** is an informed guess.

This flowchart shows how you can test a prediction.

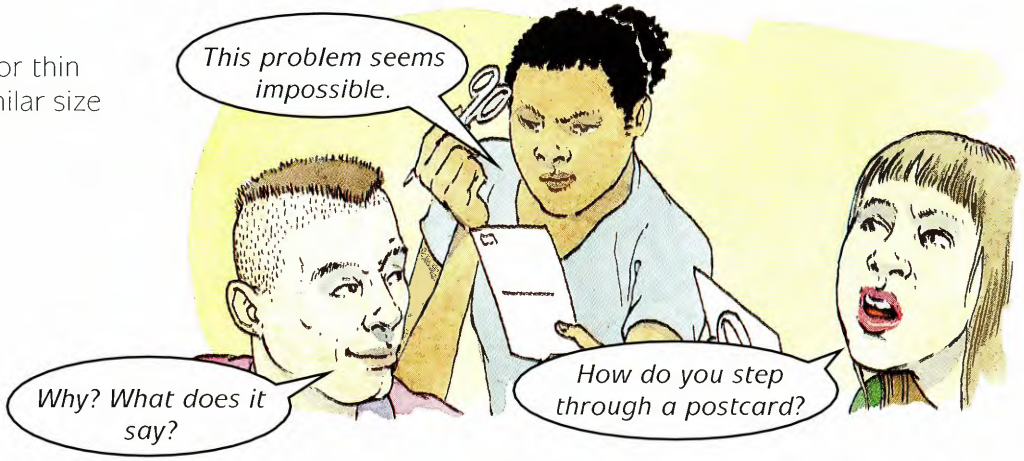


The argument/explanation is called a **proof**.

Try to turn your predictions into proofs.

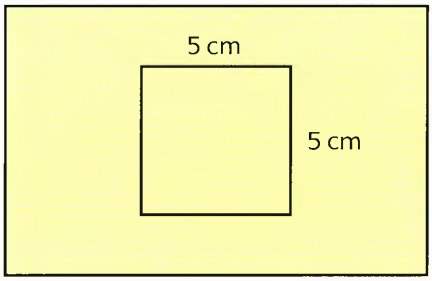
Really long curves

postcards or thin card of similar size



You can cut the postcard any way you like.

The object is to make a long, closed strip big enough to step through.



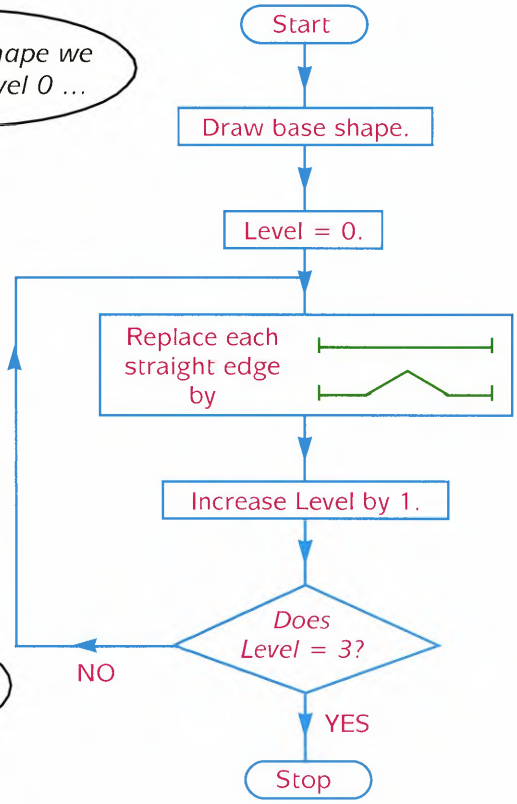
In your group, try each of these problems.

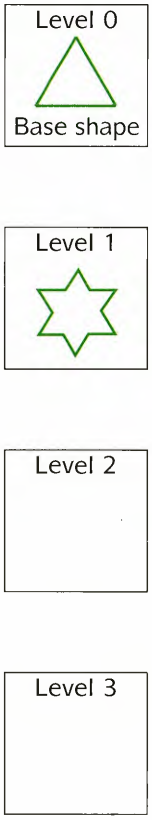
Discuss any way you think you can solve them.

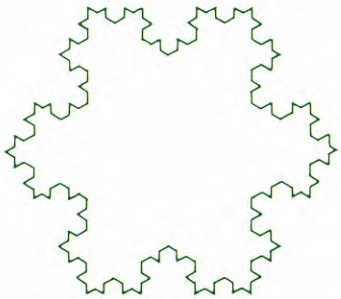
. . . Really long curves

The first problem asks for a really long shape enclosing a large area, the second asks for a really long shape enclosing a small area.

You can make a shape longer and longer by replacing each straight line with a 'crinkled' line.







A convex snowflake – Level 3

Shapes like this, where each line is replaced by a similar pattern which gives a more and more 'lacy' edge, are examples of **fractals**.

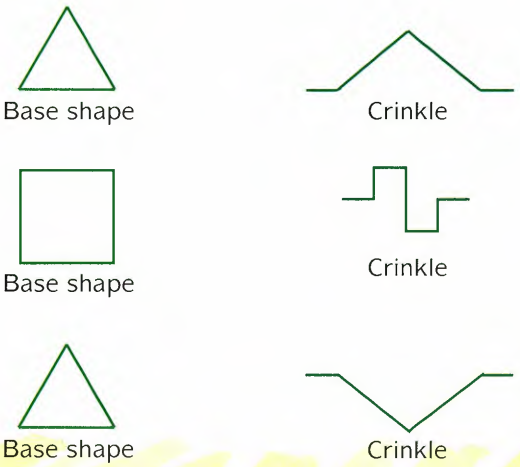
Draw the Level 3 shape by following the flow diagram.

You can only imagine the fractal because you would have to go on adding crinkles, on and on and ...

Exploring fractals

Look at the diagrams below. Use the flow diagram on page 8.

- Pick one of these base shapes.
- Draw the base shape very faintly in pencil.
- Draw in the first crinkles, faintly again.
- Then draw a final level of crinkles.



Discuss these, and other questions, in your group.

Make a wall display of your fractals. Include any information you have discovered.

. . . Really long curves

LD FRACTAL

Logo and fractals

Using 'Fractal' you can:

– choose a base shape,

– design a crinkle,

– draw your fractal.

With some base shapes and some crinkles the shape crosses itself and becomes tangled.

- Choose a base shape and a crinkle that you think won't tangle.

Draw several layers of your fractal and provide some information about it.

- Can you make 'designer fractals', which will fit a set pattern?

A fabric designer needs a fractal, based on a square, whose area is twice that of the original square.

Try to do this using 'Fractal'.

- Can you design a fractal based on any shape whose area is three quarters that of the original shape?

page 22

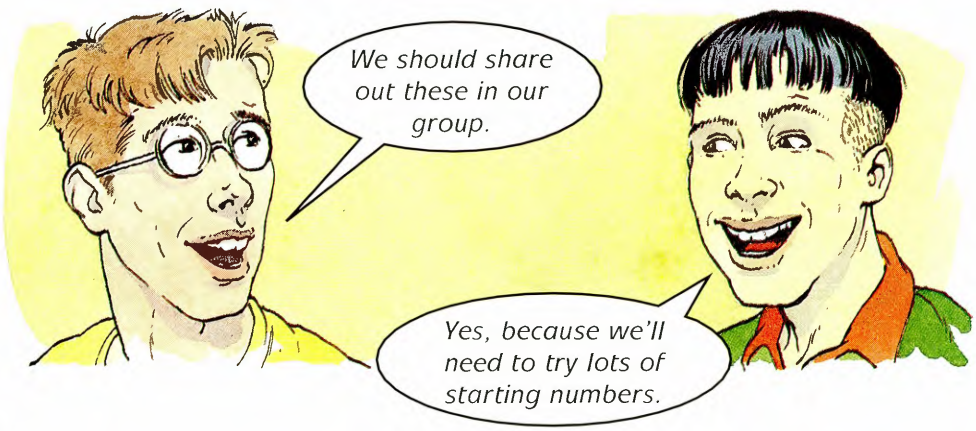
Are you sure?

calculator

or

If more than one of these pieces of equipment is available, read 'Choosing the mathematical tool' on page 13, before deciding which to use.

Try out procedures (a) to (f) to test the comments beside them.



It looks as if it gets closer and closer to 0.

(a) Enter any positive number.
Keep pressing the sin button.

(b) Enter any positive number.
Keep pressing the x^2 button.

It looks like it gets bigger and bigger.

It settles down to 1.

(c) Enter any positive number.
Keep pressing the $\sqrt{}$ button.

(d) Enter any positive number.
Keep pressing the $1/x$ button.

This moves between two numbers.

This settles down to move between two numbers.

(e) Enter any positive number between 0 and 1.
Press the x^2 button and subtract 1.
Keep doing this.

(f) Enter any positive number between 0 and 1.
Press the x^2 button, multiply by 2 and subtract 1.
Keep doing this.

It gets closer and closer to 0.

Note: Not all the above comments are true statements.

. . . Are you sure?

Make a copy of this table.

Enter (a), (b), (c), (d), (e) and (f) in the appropriate column.

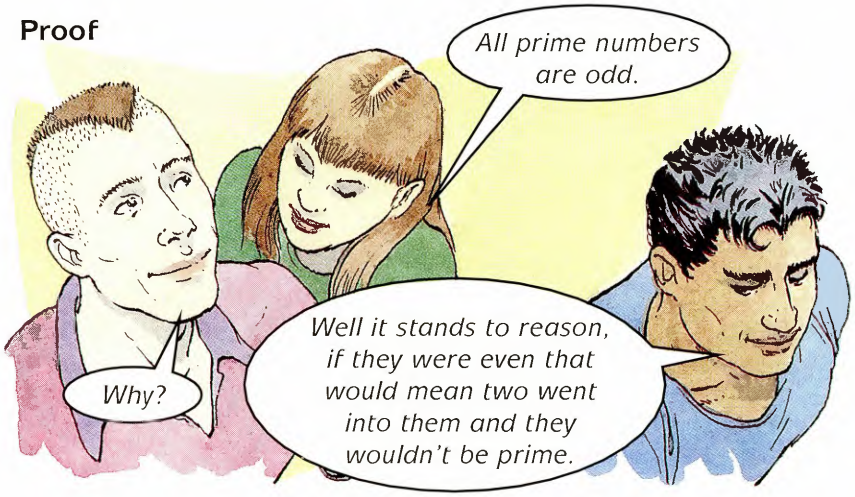
Definitely false	*Not certain*	*Definitely true*

Compare your results with the rest of your group.

Discuss any differences or similarities.

- Choose one of the true statements and try to explain exactly why it has to be true.

Proof



A proof has to convince you for every single possible case.

If you can think of just one number that the proof above does not work for – then you have disproved it.

- Are all prime numbers odd?

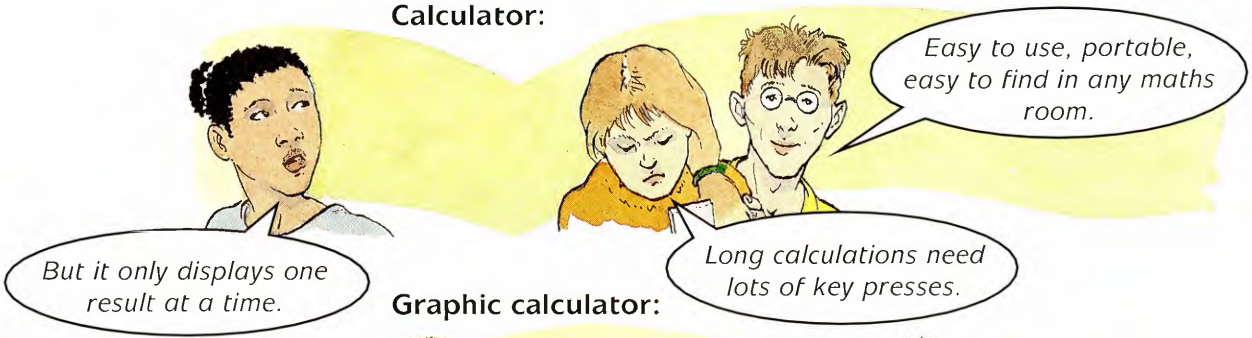
Choosing the mathematical tool

The word **iteration** is used to describe an operation which is repeated over and over again.

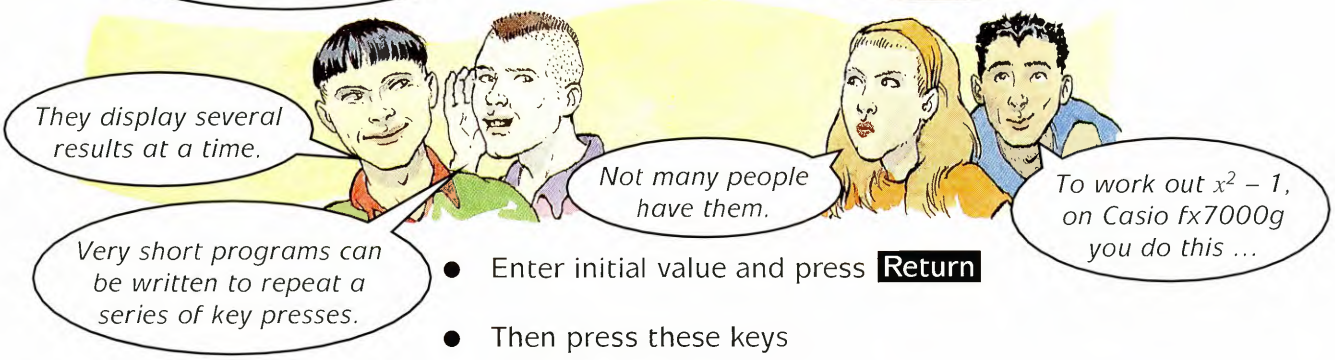
Here are some of the advantages and disadvantages of possible tools for iteration.

You will need to decide which you prefer to use from the tools available to you.

Calculator:



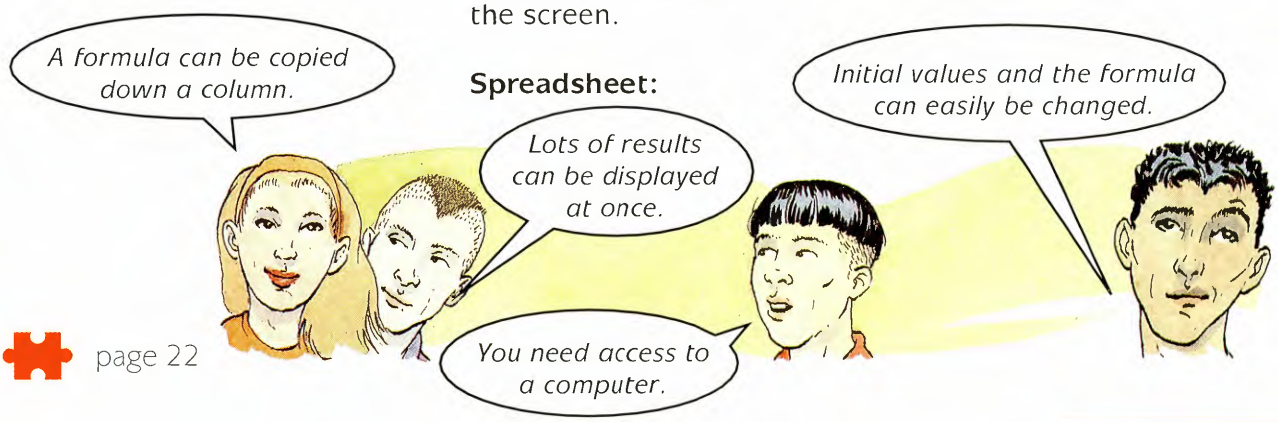
Graphic calculator:



- Enter initial value and press **Return**
- Then press these keys

  Ans x^2 **−** **1**
- Keep pressing **Exe** and the values will rapidly scroll up the screen.

Spreadsheet:



page 22

Towards chaos

The activities in this Theme have been chosen to introduce you to some of the ideas behind these pictures.

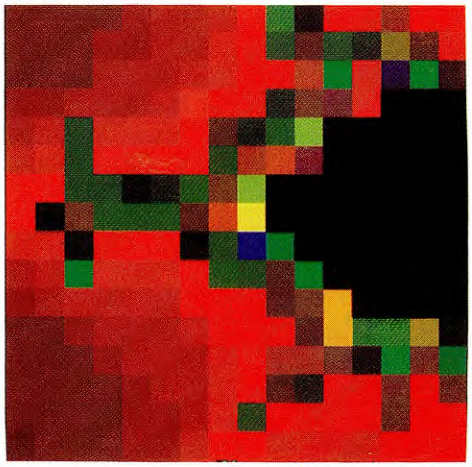
What do the colours mean?

Like the colour graphs you have drawn, the overall picture is made by

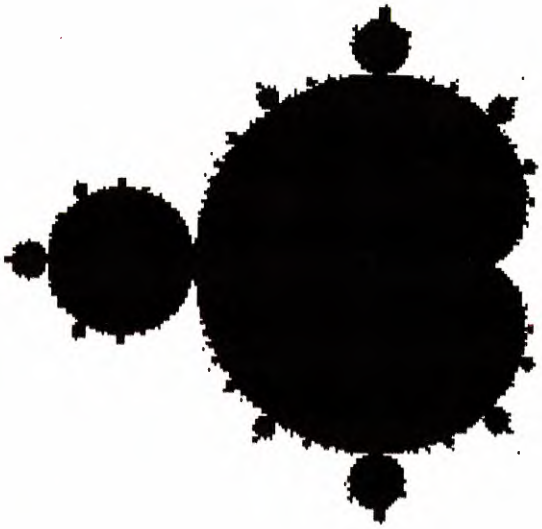
- choosing one small square in the grid,
- making a mathematical calculation based on the coordinates,
- deciding on a colour,
- adding the colour to the grid.

In this picture a coarse grid is used and you can see the blocks of colour.

(*1*)



(2)



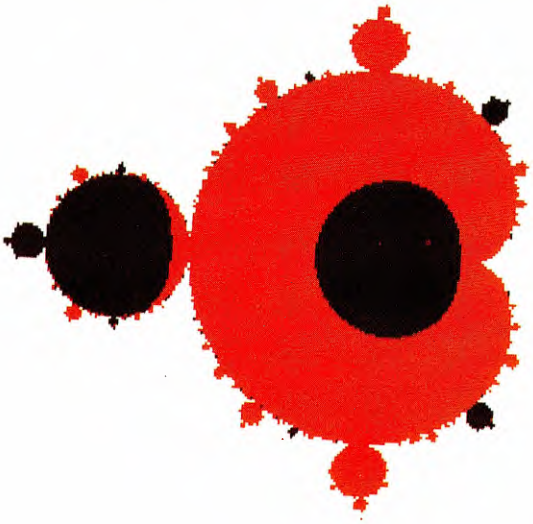
In this picture, the colour code used is:
- white if the numbers in the calculation tend to infinity,
- black if they don't.

Choosing different colour codes can make the pictures look different even though the same calculation is used.

For the picture below, the colour code, after 100 calculations is:

- white if the number is greater than 2,
- red if the number is greater than 0.5 but less than 2,
- black if the number is less than 0.5.

(3)

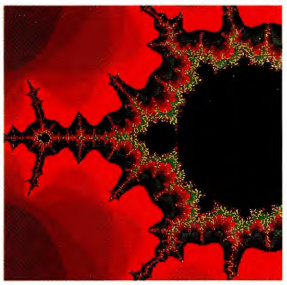


. . . Towards chaos

What sort of calculation gives such complicated pictures?

To make this picture, an iteration like that in the activity on page 11 is needed. For each square of this grid not just one calculation is made but hundreds and hundreds.

(4)

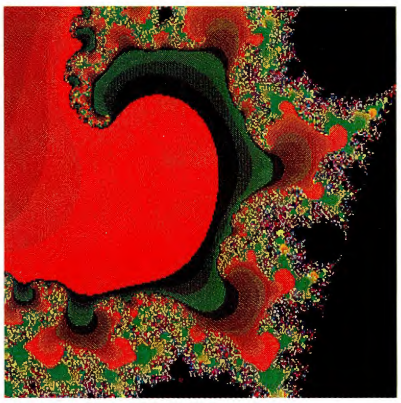


The actual equation used is only slightly more complicated than the $2x^2 - 1$ that was used in the activity on page 11.

For years and years scientists and mathematicians used equations like this and did not realise what complicated and chaotic results they could get.

The change was brought by the work of Lorenz and Mandelbrot (after whom the pictures are named) and the advent of powerful computers to do the calculations.

(5)



So if it is not the equation that is complicated why are the pictures so complicated?

Discuss this in your group.

What use are chaotic pictures?

If the pictures really were complicated, chaotic, and unpredictable, they wouldn't have attracted much attention.

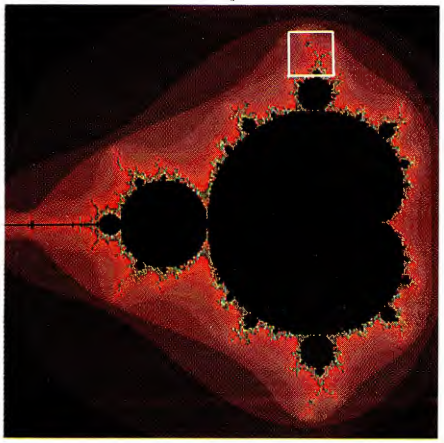
Instead they are beautiful and although there are no simple patterns, there is one important feature.

Like fractals, zooming in on these pictures shows how complicated they are ...

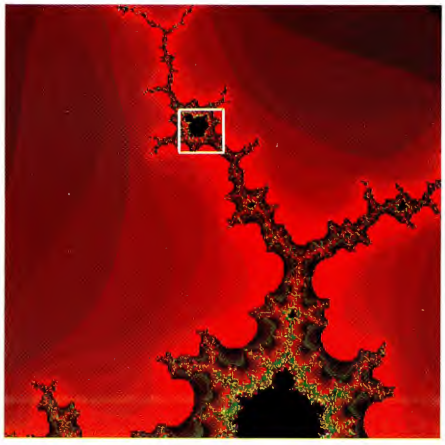
... the same shapes turn up again and again.

Zoom in on this square to get the next picture

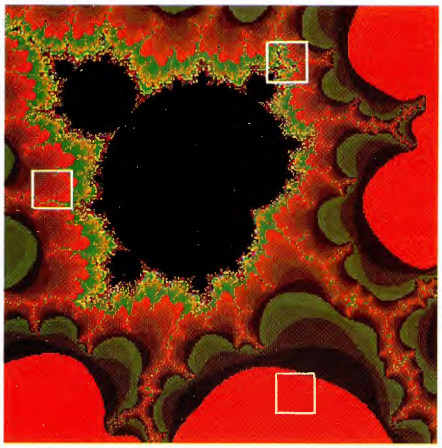
(6)



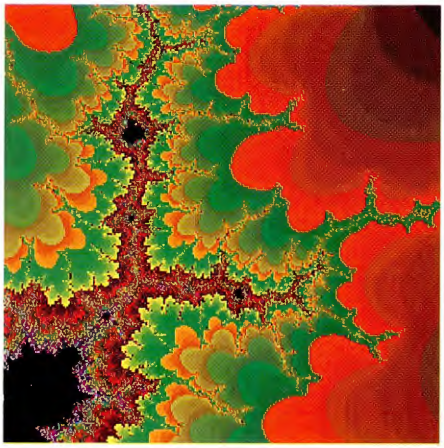
(7)



(8)



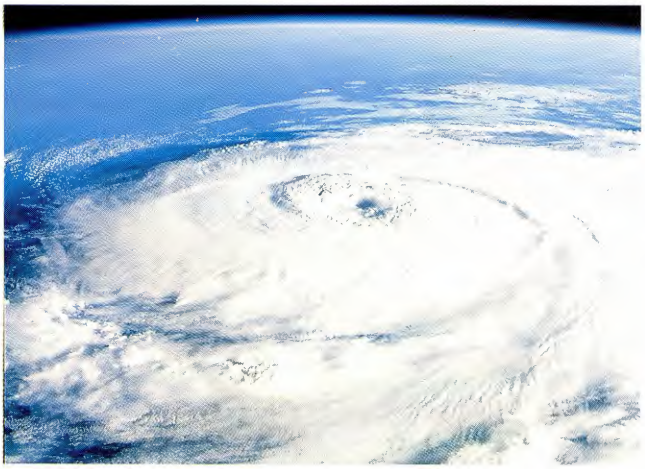
(9)



Which of the three zoom boxes in (8) gives the final picture, (9)?

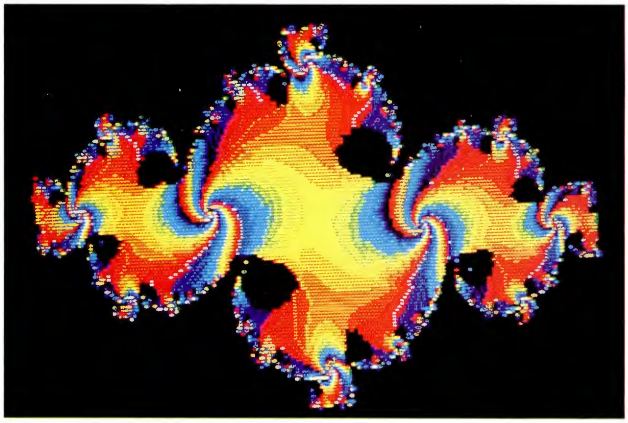
. . . Towards chaos

These ideas are called the **ideas of chaos**.
The ideas of chaos are being used in biology, medicine and meteorology. What seemed a complicated mathematical idea is turning out to have practical uses.



Chaos theory is used as a means of modelling processes such as weather systems

Turbulence in a fluid is an example of a chaotic system



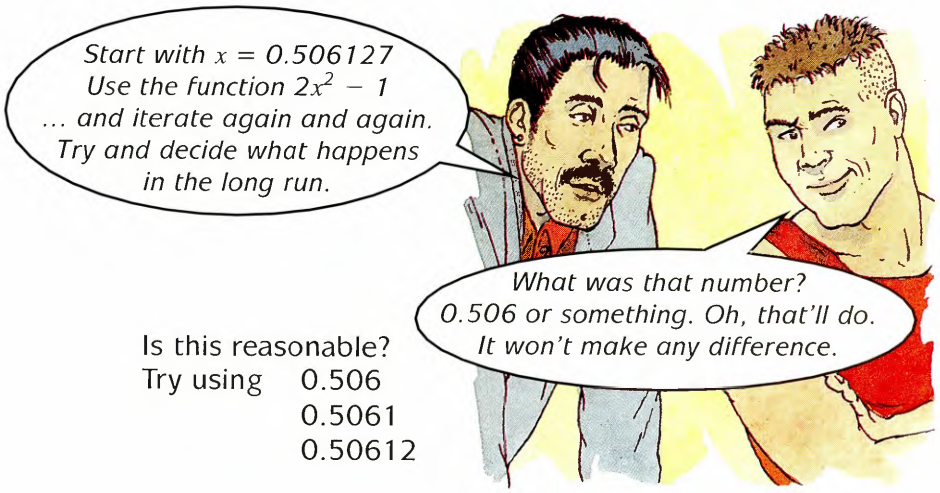
'Chaotic attractors' – a computer graphics image generated using complex numbers

Almost all the mathematics you learn in school was developed centuries ago. 'Chaos' has been developed in the second half of the 20th century and is a current research area for scientists and mathematicians.

The butterfly effect

calculator or graphic calculator

Read this conversation carefully:



Is this reasonable?
Try using 0.506
0.5061
0.50612

What happens if you iterate for many times?

Now try it for some functions and some starting values of your own.

You will discover that the long-term effects are not:

- safe,
- predictable,
- obvious.

Results can vary a great deal because of a tiny change in the starting conditions.

They are CHAOTIC

A brief history

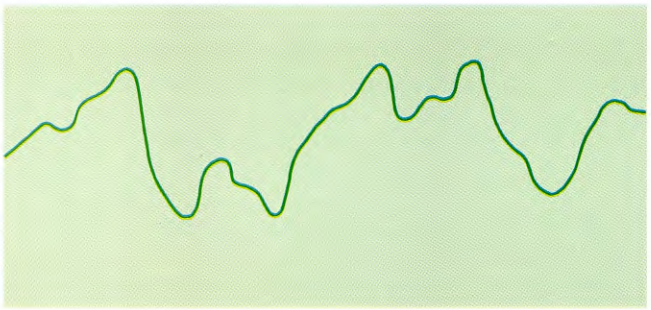
The calculator experiment above using two slightly different starting values is very close to an experiment by Edward Lorenz in 1960 which started interest in chaos.

Edward Lorenz entered some formulae into a computer.

He was trying to predict the weather, the wind speed and the temperature from given starting conditions.

. . . Towards chaos

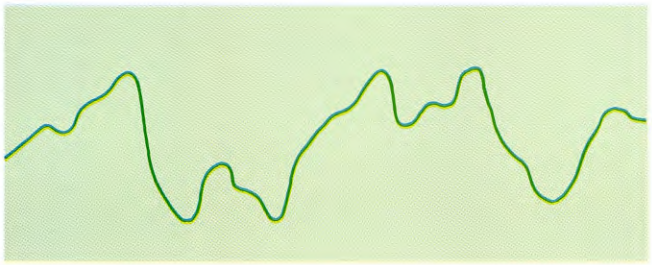
The computer produced a printout like this:



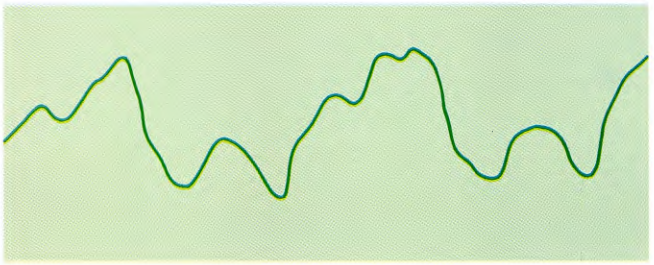
Lorenz wanted to explore this weather pattern further so he typed in a starting value of 0.506 and went off to get a cup of coffee.

When he returned, he had two printouts.

Original



New



They should have been the same.

What would you have done?

Most people would have scrumpled up one or other of the printouts.

But Lorenz put one printout on top of the other. He realised that, although he had typed in 0.506 to produce the second printout, the number used by the computer for the first printout was 0.506127. The two printouts started from very slightly different positions.

A tiny change in initial conditions made an enormous change in the long-term prediction.

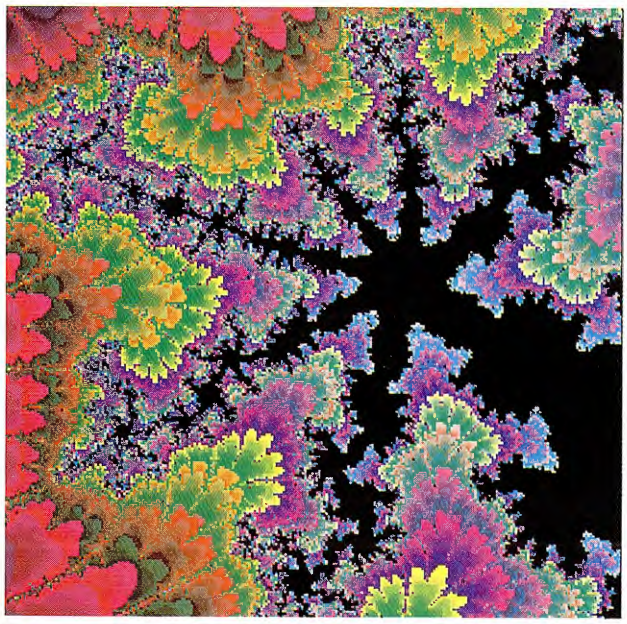
A picturesque way of putting this is that:

A butterfly flapping its wings in one country, can change the initial weather conditions so that a hurricane will blow up in another.

The connection with the pictures is that:

If this is the picture you get using starting values at intervals of 0.001

(10)



then you can always zoom in on a more complicated picture by choosing starting values at intervals of 0.0001.

Now it is your turn.

Iterating $x^2 - 1$ gives order for any starting value of x.

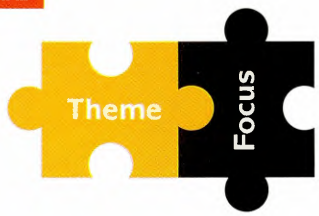
Iterating $2x^2 - 1$ gives chaos.

- What happens for $(1.1)x^2 - 1$ and $(1.2)x^2 - 1$?
- Where does order change to chaos?

Investigate these two questions with the rest of your group.

Try to reach some conclusions.

page 22



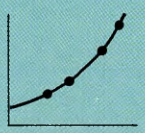
Here are some kinds of maths you may have used in this Theme.

You can find out more about them in these Focus book units.

Formulae and equations

$p = 2l + 2b$ Algebra **E** 2

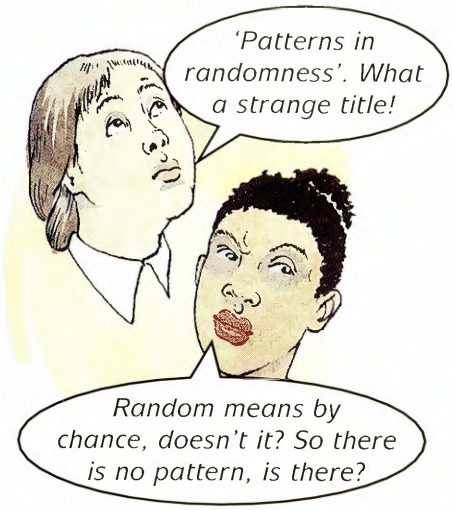
Graphs



Algebra **E** 2, 5

Number operations

$\left(+ \; \sqrt{\;} \; \times \; - \; \div\right)$ Algebra **E** 7



PATTERNS IN ... RANDOMNESS

Before you start this Theme discuss with a partner what is meant by the word **random** ... You might think that randomness means having no pattern. In this Theme you will explore that idea. There are four sections.

section	aspect	page
Reaction times	using the computer to investigate reaction times	24
Matching birthdays	discovering the chance of two people in your class having the same birthday	31
The colour machine	making a machine which generates random colours	34
This dice is unfair!	discovering if a dice is biased	41

All four activities will require you to use a computer and the Logo 2000 software.

There are more ideas and hints on page 47

References to Focus books are on page 48.

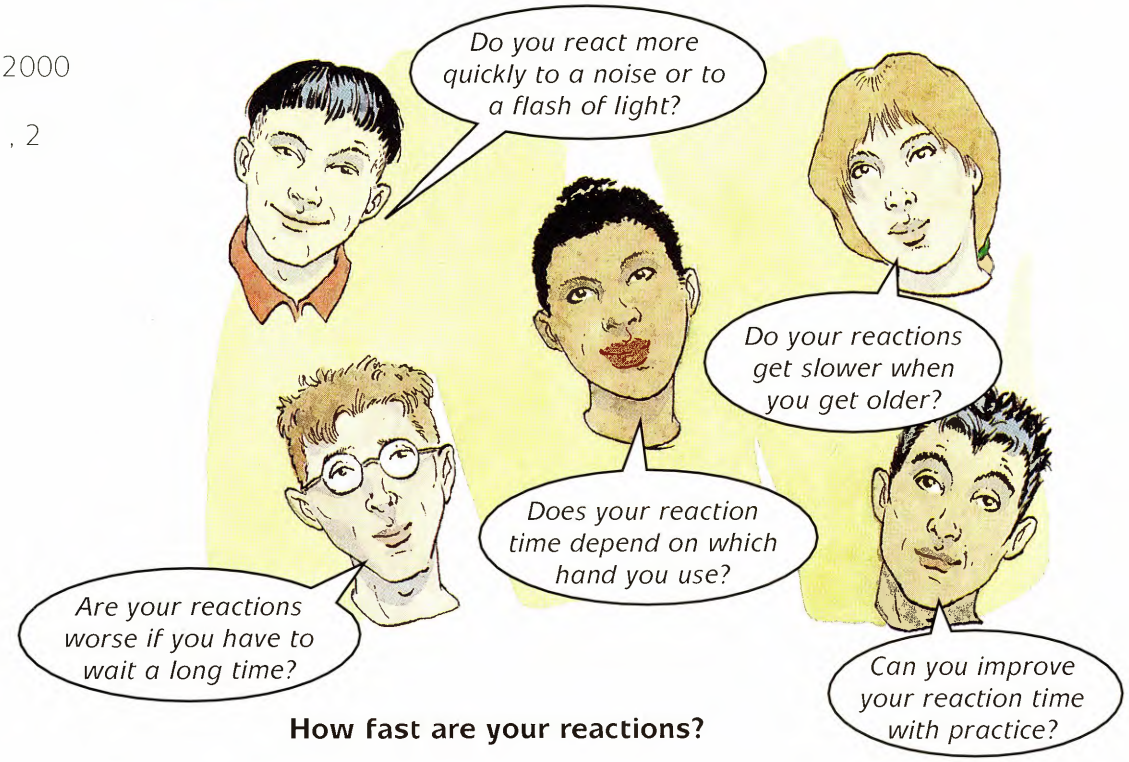
Symbols in the margin

- Special things you *will* need
- Special things you *may* need
- More ideas
- Link page
- Worksheet reference
- Work for graphic calculator here
- Work for computer here (screen may show one of these: D Database G Graph plotter S Spreadsheet L Logo)
- LD Logo 2000 disk
- LN Logo 2000 documentation
- LP LogoPack

Reaction times

Logo 2000

1, 2



How fast are your reactions?

Have you ever wondered how fast your reactions are? This series of experiments lets you find out.

First, you will find out how quickly you can react to a flash of light.

- You will need LogoSheet and LogoPlotter. Load these in the usual way.
- The reaction test also uses some procedures in a file called 'React' on the disk. To use these procedures, type:

  LOAD "REACT
- You will also need the spreadsheet file called 'Reacsheet'. To use this type:

  LOADSHEET "REACSHEET

  You may need to type **START** to display a spreadsheet and a set of axes.

- When you run the experiment, LogoSheet will use five of the cells for your reaction times.

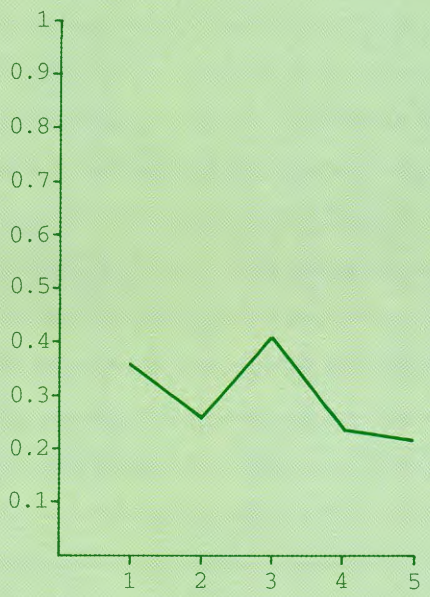
To try this out type:

DO

Just press a key each time you see a flash. There will be five flashes.

LogoSheet values

	A	B	C
1	Goes	Times	
2	1	0.36	
3	2	0.26	
4	3	0.41	
5	4	0.24	
6	5	0.22	
7	Mean time	0.3	
8			
9			



If you want a fresh graph, then use **START** again.

. . . Reaction times

Some experiments to try

Are you quicker than a friend?

Try the reaction experiment with a partner. Have five goes each.

How can you decide who has the faster reactions?

Can you improve with practice?

You and your partner try the experiment a few times using the same graph.

Are you improving with practice? You could record your mean reaction times and see if these are going down.

Left hand versus right hand

Use the same set of axes for the graphs and decide which is the better hand to use in reaction tests.

Fold your arms

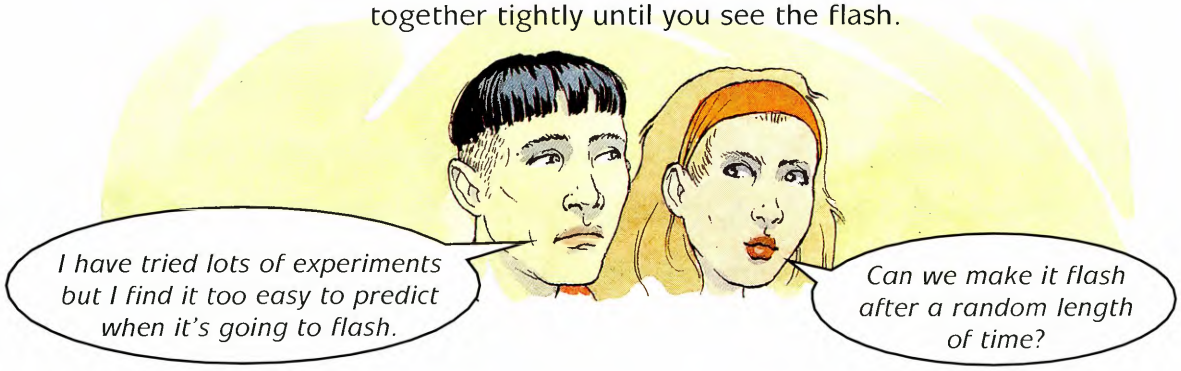
Wait with your arms folded until you see the flash. How fast can you unfold them and get to the keyboard?

Tense your muscles

When muscles are tense they usually react more slowly. This is one reason why sprinters warm up just before the start of a race.

Try the reaction test again.

This time, try making your arm muscles tense by pushing your hands together, palm to palm. Keep them pushed together tightly until you see the flash.

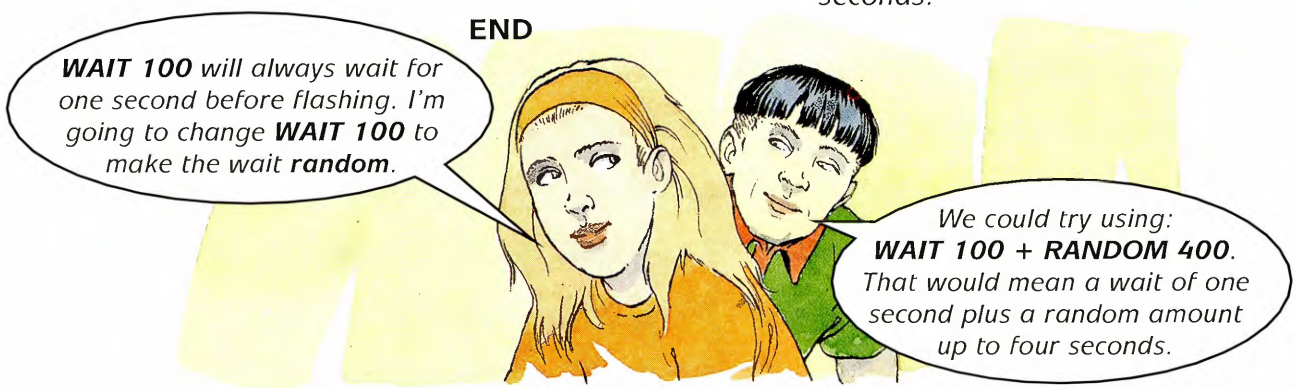


The procedure which measures the time taken for you to press a key is called **REACT**. You can edit this procedure to try out different ideas.

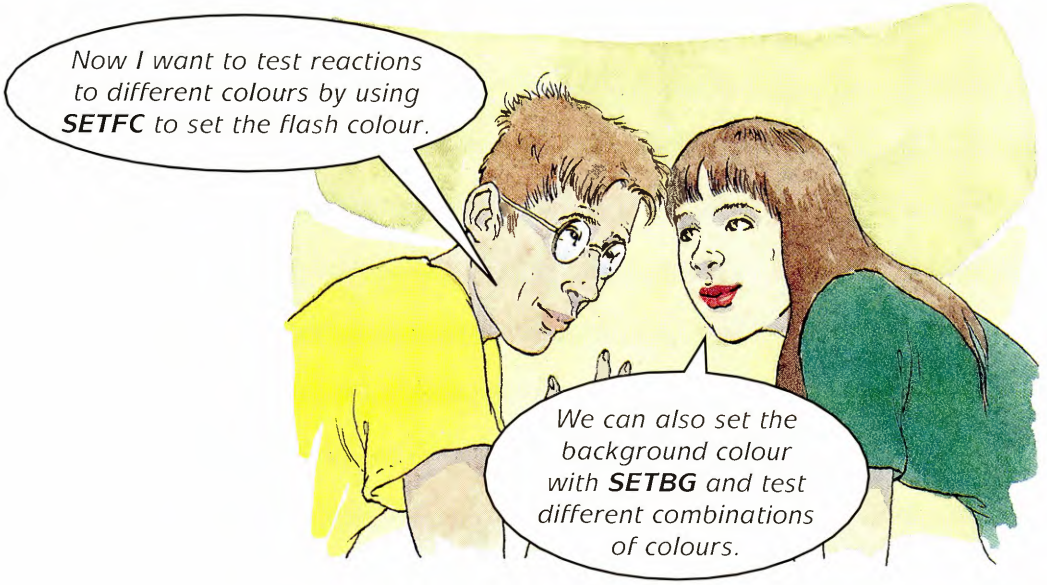
```
TO REACT
WAIT 100                    will wait for 1 second.
IF KEY? [CLEARKEYS]         looks to see if a key is already
                            pressed.

FLASH
ZEROTIME                    starts the clock.
MAKE "KEY RC                waits until a key is pressed.
OP TIME/100                 outputs the time taken in
                            seconds.

END
```



. . . Reaction times



SETFC	*controls the colour of the flash.*
SETFC 3	*will give a yellow flash. Flash colour codes are 1 red, 2 green, 3 yellow, 4 blue, 5 magenta, 6 cyan, 7 white.*
PR FC	*will print the current flash colour.*

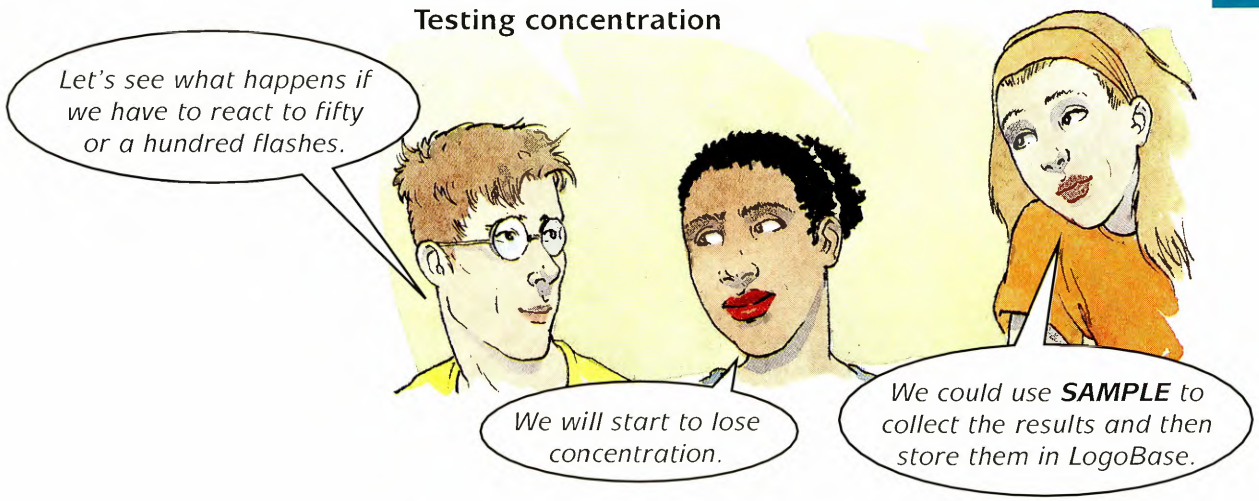
Try some of these ideas in your group.

- Distract the person with the wrong colour flashes.
- Try testing the left and right eye separately.
- Try putting **SETFC** into the **REACT** procedure to make a random flash colour.
- Try using a random flash colour but penalise a reaction to the wrong colour.

There is a procedure called **MEANLINE** which will draw a horizontal line for you at the place of the mean of a set of results.

This can be done each time you need it by typing **MEANLINE** or the procedure name can be placed in the formula sheet in cell **B8**. In this case it will appear every time the experiment is run.

Testing concentration



SAMPLE	*gets reaction times and stores them in a list.*
PR SAMPLE 10	*will print out a list of 10 reaction timings.*

LogoBase can be used to collect the data produced by **SAMPLE**.

- Load LogoBase if it is not already loaded.
- Give your data file a name such as 'Times'.

  USEDF "TIMES

- Tell LogoBase the field names, for example your field names might be GO and TIME.

  SF [GO TIME]

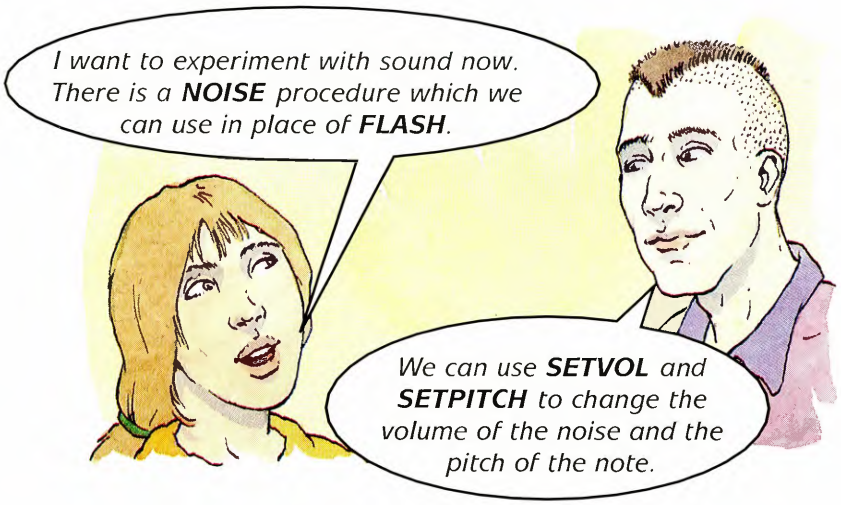
  (**SF** is short for **SETFIELDS**.)

- Add 50 records to your data file by typing:

  ADDRECS PAIRS NUMBERS [1 50] SAMPLE 50

NUMBERS [1 50]	*produces a list of numbers from 1 to 50.*
PAIRS	*produces ordered pairs from two lists.*
ADDRECS	*adds the new records to the data file.*

A sound reaction



SETVOL *controls the volume of the sound.*

SETVOL −15 *gives the loudest noise.*
SETVOL −1 *gives the softest noise.*

This rather strange choice of volume range matches the range available in the Logo ***SOUND*** *command.*

SETPITCH *controls the pitch of the note.*

SETPITCH 255 *gives the highest note.*
SETPITCH 0 *gives the lowest note.*

1, 2
page 48

For some more ideas dealing with time, look at Worksheets 1 and 2. These two worksheets need to be used together.

Matching birthdays

Logo 2000

Do you think there are two people in your class with the same birthday?

With a partner, find out the birthdays of about 30 people – the year does not matter.

Did any two people have the same birthday?

Is that what you expected?

How can you decide whether your result is typical?

You will need to look at many classes and see how often you get a matching birthday.

However, first it may help to make a guess.

Estimate how many classes (out of 100) would, in your opinion, have at least one matching birthday.

Looking at lots of registers would take too much time but you can make a computer do much of the work.

- Load Logo and LogoBase in the usual way.

You are going to generate 30 random birthdays and put them into a LogoBase data file.

You will then be able to inspect the birthdays to see if any two match.

This will be a lot quicker than looking at the registers.

. . . Matching birthdays

How to create a random birthday

Apart from leap years, there are 365 days in the year. A birthday can be represented by a number from 1 to 365.

In Logo, the **RANDOM** command produces a random number.

For example, **RANDOM 10** produces a random number between 0 and 9.

To produce a random number between 1 and 10, we would need **1 + RANDOM 10**.

- What would you use to get a random number between 1 and 365?

How to put a random birthday into LogoBase

- Tell LogoBase that the name of the data file is 'Birthdays' and that the name of the field is DATE.

```
USEDF "BIRTHDAYS
SETFIELDS [DATE]
```

- Create a random record with the command:

```
ADDREC LIST 1 + RANDOM 365
```

The command **ADDREC** adds a record to the data file. The record to be added must be a list containing the random birthday.
Check the record by typing:

```
TA
```

- You will need 30 birthdays altogether, so type:

```
REPEAT 29 [ADDREC LIST 1 + RANDOM 365]
```

and use **TA** to check the class of random birthdays.

How to check for matching birthdays

The 'Birthdays' data file now contains 30 random birthdays. You could look at them by showing the records in a table. However, it would be difficult to spot any matching numbers.

By sorting the records first it becomes easy to spot any matches.

- Sort the records and look for any matches.

TR SORTALL "DATE

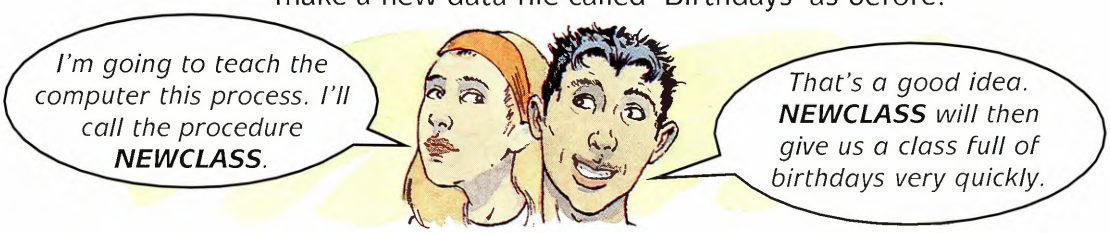
- Repeat this process a number of times to see how often you get matches.

To repeat the process you should

delete the old data file before creating a new one by typing:

DELDF

make a new data file called 'Birthdays' as before.



- Does your set of results suggest that your original estimate was wrong?

If so, make a new estimate of how many classes out of 100 would have a matching birthday.

Discuss your conclusions with a partner. Try to persuade your partner that you are right.

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The colour machine

Logo 2000

You will need a computer, Logo and LogoSheet for this activity.

You are going to design a game for a friend to play.

The game is played on a computer using LogoSheet.

- Load Logo and LogoSheet in the usual way and then type:

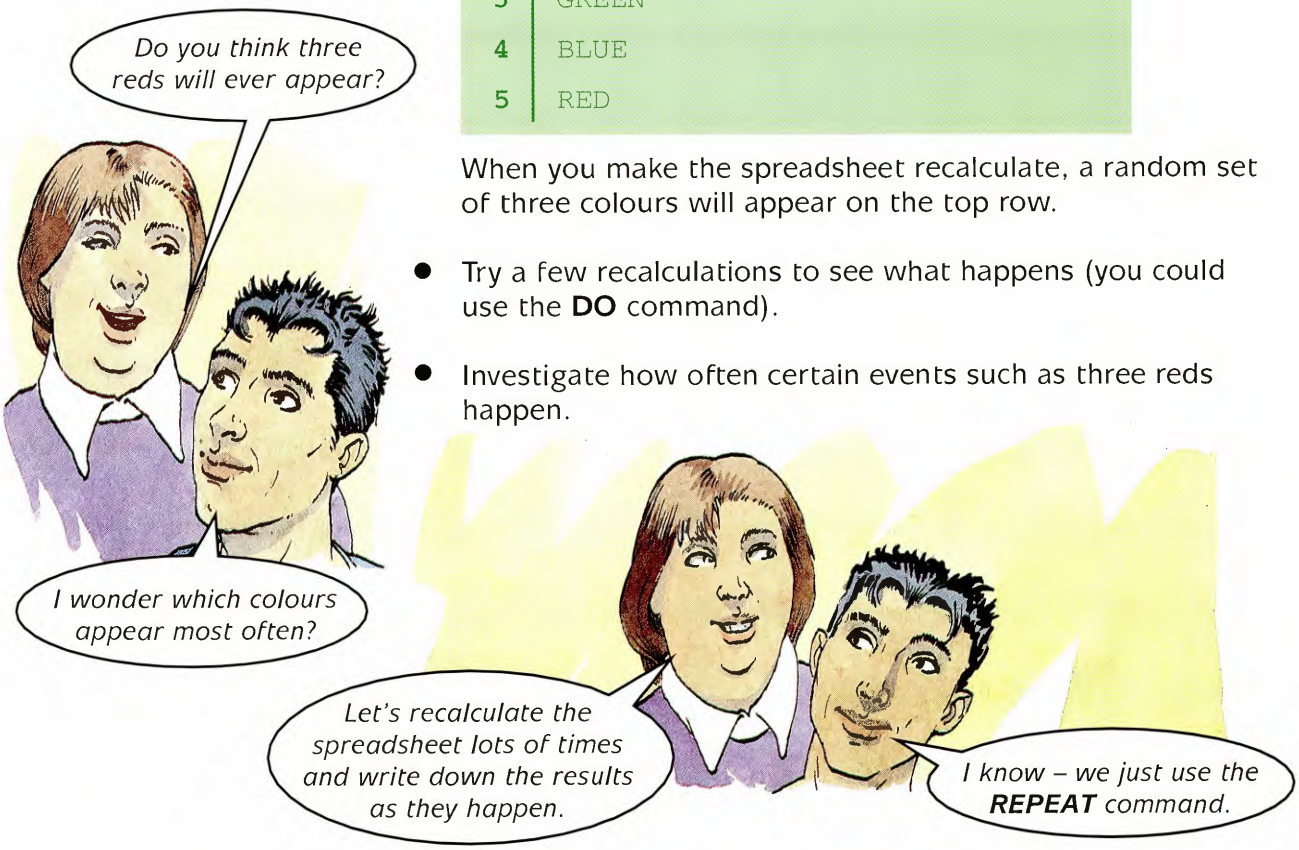
LOADSHEET "MACHINE
LOAD "MACHINERY

LogoSheet values

	A	B	C
1	RED	BLUE	GREEN
2			
3	GREEN		
4	BLUE		
5	RED		

When you make the spreadsheet recalculate, a random set of three colours will appear on the top row.

- Try a few recalculations to see what happens (you could use the **DO** command).
- Investigate how often certain events such as three reds happen.



LogoSheet values

	A	B	C
1	RED	BLUE	GREEN
2			
3	GREEN		
4	BLUE		
5	RED		

```
? REPEAT 30 [DO HALT]
```

HALT will make the spreadsheet pause until you press a key after each recalculation.

This procedure was loaded in as part of the 'Machinery' file.

The first column from **A3** downwards tells you the colours that might be chosen.

You can imagine a green ball, a blue ball and a red ball in a bag. Whenever the spreadsheet recalculates it chooses one of these balls and writes its colour in the top row.

The ball is then replaced in the bag before the computer chooses another one.

- Change the colour in **A3** (or below) and see what happens when you do some recalculations.

. . . The colour machine

LogoSheet values

	A	B	C
1	RED	BLUE	GREEN
2			
3	GREEN		
4	BLUE		
5	GREEN		

```
? REPEAT 30 [DO HALT]
```

Now you can design a game for a friend to play.

What colours do you want in cells **A3**, **A4** and **A5**?

What are the winning lines?

How much will you pay out on each of these winning lines?

Remember you don't want to make a loss but you need to make people want to play the game.

Challenge someone to play the game lots of times and see whether you make a profit.

You will need to redesign the game if nobody wants to play it or if you lose too many counters.

The next activities give you some help on how to make further use of the spreadsheet. Perhaps you already have some ideas.

How to record the stakes

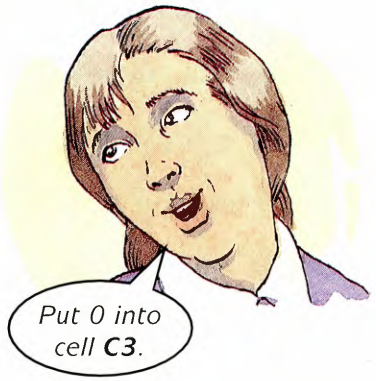
Each time your partner plays, he/she pays you one counter.

You can tell the spreadsheet how to record the total number of counters your partner has paid.

The number of counters staked so far will appear in cell **C3**.

LogoSheet values

	A	B	C
1	BLUE	BLUE	GREEN
2			
3	GREEN		0
4	BLUE		
5	GREEN		



LogoSheet formulae

	A	B	C
1	CHOOSE	CHOOSE	CHOOSE
2			
3			C3 + 1
4			
5			

The Formula Sheet already contains the word **CHOOSE** in cells **A1**, **B1** and **C1**.

This tells LogoSheet to choose a random colour from those in the first column.

CHOOSE is a procedure that was loaded into the memory as part of the 'Machinery' file.

. . . The colour machine

How to record the payout

Whenever there is a winning line on the top row, you will have to pay out. You can tell the spreadsheet to record the total number of counters paid out.

When you first started you loaded in a file called 'Machinery'. One of these procedures is called **WINS**. You can change this to include whatever winning lines you want.

The **WINS** procedure looks like this:

```
TO WINS
OP [[RED RED RED][GREEN GREEN GREEN][BLUE BLUE BLUE]]
END
```

Now there are three winning lines: RED RED RED,
GREEN GREEN GREEN,
and BLUE BLUE BLUE

- Edit the **WINS** procedure.

  For example, if you only wanted three reds to win, the **WINS** procedure would look like this:

  TO WINS
  OP [[RED RED RED]]
  END

- Now you need to tell Logo how much each of these winning lines pays out.

  When you loaded the 'Machinery' file, you loaded a procedure called **PRIZES**. It looks like this:

  TO PRIZES
  OP [5 5 5]
  END

- Edit the **PRIZES** procedure.

  For example, if you want three reds to pay out two counters and this is the only winning line, the **PRIZES** procedure would look like this:

  TO PRIZES
  OP [2]
  END

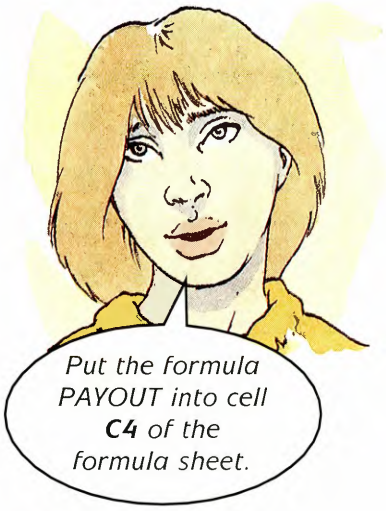
  Now you can record the payout on the spreadsheet.

When you loaded the 'Machinery' file, you loaded a procedure called **PAYOUT**.

This procedure uses the **WINS** and the **PRIZES** procedures to calculate the payout whenever you recalculate.

LogoSheet formulae

	A	B	C
1	CHOOSE	CHOOSE	CHOOSE
2			
3			C3 + 1
4			PAYOUT
5			



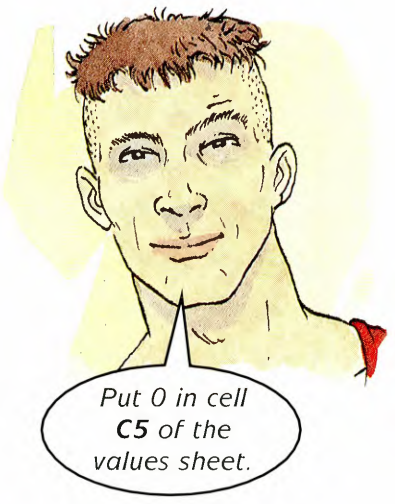
How to record the profit

The spreadsheet can record your profit to help you decide if your game is well designed.

The profit so far will appear in cell **C5** of the values sheet.

LogoSheet values

	A	B	C
1	BLUE	BLUE	GREEN
2			
3	GREEN		0
4	BLUE		0
5	GREEN		0

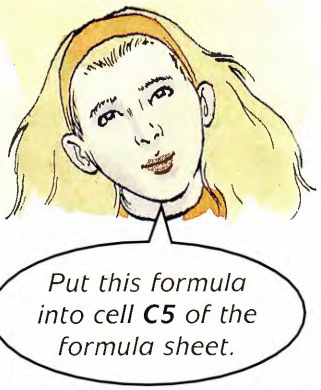


. . . The colour machine

The new profit each go will be the old profit plus the stake less any payout.

LogoSheet formulae

	A	B	C
1	CHOOSE	CHOOSE	CHOOSE
2			
3			C3 + 1
4			PAYOUT
5			



After you have designed your machine, test it by doing lots of recalculations. Does your game make a profit for you? Will it attract customers?

When you have a good design, play the game in your group.

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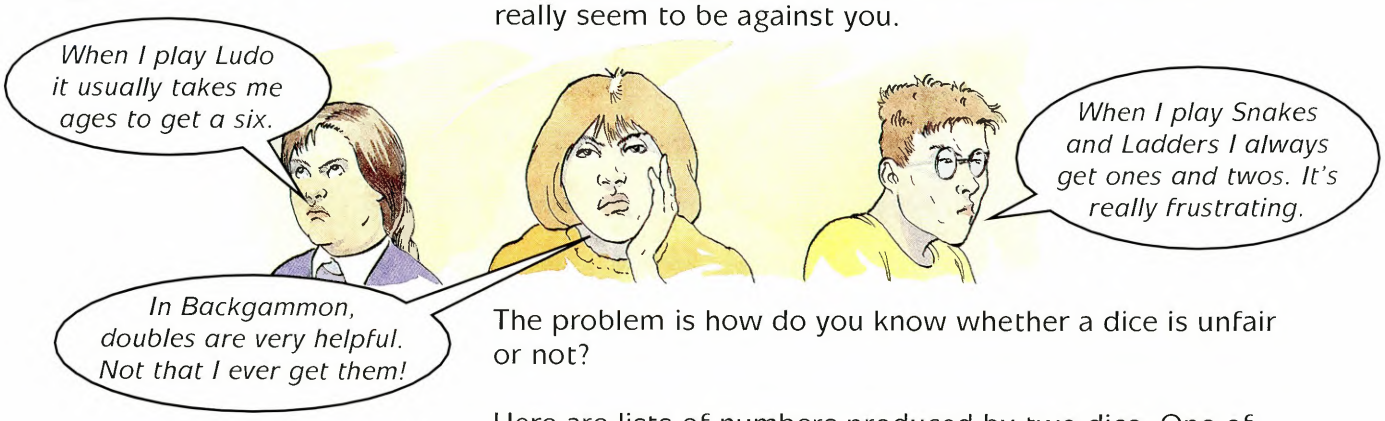
page 48

This dice is unfair!

Logo 2000

Lots of board games use dice and sometimes the dice really seem to be against you.



The problem is how do you know whether a dice is unfair or not?

Here are lists of numbers produced by two dice. One of them is unfair.

Green	3 4 1 2 1 3 3 4 6 4
Yellow	2 6 4 3 5 1 1 3 1 5

Discuss with a partner why it is so difficult to tell which is the unfair dice.

What would you do to find out?

It would help to roll the dice many times but this can be boring. The computer can help.

- Load Logo in the usual way and then type:

  LOAD "UNFAIRDICE

- You have loaded some procedures, one of which is called **ANYOF**. This procedure chooses a number at random from a collection of numbers.

  For example, type:

  PR ANYOF [1 2 3 3 3 4 5 6]

- Type this a few more times and compare your results.

. . . This dice is unfair!

In the example on page 41, the green dice is unfair because it is more likely to roll a 3.

The **GREEN** procedure below uses **ANYOF** to simulate a fair dice.

```
TO GREEN
OP ANYOF [1 2 3 4 5 6]
END
```

- Create the **GREEN** procedure and test it out.
- Write another procedure called **YELLOW** that produces random numbers from an unfair dice.
- Print out 10 numbers from each dice and see if a partner can tell which is the unfair dice.

The great dice challenge

The computer is going to set a challenge for you by creating two dice. You have to work out which is the unfair dice.

You already have a procedure called **MAKEDICE** which was loaded as part of the 'Unfairdice' file.

This procedure creates a red and a blue dice.

- Type **MAKEDICE**.

You now have two procedures, **RED** and **BLUE**.
One of these represents a fair dice and the other an unfair dice.

- To see the results of rolling the blue dice 10 times, type:

```
REPEAT 10 [ PR BLUE ]
```

Do the same for the red dice. Can you tell which dice is unfair?

You are going to use LogoBase to store throws from the red and blue dice so that you can compare them.

- Load the Logo 2000 software in the usual way.
  You will need LogoBase and LogoPlotter.

- Tell LogoBase that the name of the data file is 'Dice' and that the fields are called ROLL, BLUE and RED.

  USEDF "DICE
  SETFIELDS [ROLL BLUE RED]

  ROLL is the name of a field which will contain the roll number.

  The first roll of each dice will have number 1 and the tenth roll will have number 10, etc.

- Create the following procedure which adds one record into the DICE data file.

  TO ROLL
  ADDREC (LIST CA +1 BLUE RED)
  END

  CA is short for count all – the number of records already in the data file. So **CA +1** gives the next record number.

- Use this procedure to generate 20 records by typing:

  REPEAT 20 [ROLL]

- Putting all these records in a table will help you check that you understand them.

- Type **TA**

  Can you work out which is the unfair dice?

- In your group, discuss how you can discover the unfair dice.

  Perhaps you need some clues to help you.

  If so, you will find some ideas on the next three pages.

. . . This dice is unfair!

Here are some clues to help you to play the detective.

You should work through all the clues in order to get as much information as possible before making a decision.

Clue 1: Mean average

If one dice is weighted towards higher or lower numbers, you might be able to find a difference between the mean averages.

- Find the mean average using the **MEANOF** command.

Your problem is trying to decide whether there is a big enough difference to reach a conclusion.

Clue 2: Frequencies

One of the dice may have rolled more sixes than the other.

- Find the frequencies of each number for each dice using the **FREQ** command.

For example, you can find how often the blue dice rolled a 1 by typing:

PR FREQ 1 "BLUE *print the frequency of 1 in the blue field.*

Some frequencies will be higher than others even when the dice is fair.

Are your results so different that you know which one is unfair?

Clue 3: Distribution

It may help to see a chart from which you can easily compare the frequencies. Here is how you can produce a chart for the blue dice.

- You need to tell LogoBase to count the dice scores in the BLUE field. We can put the results into a table with headings:

  Score and *Frequency*.

  TALLY "BLUE "Score "Frequency

  To see the results, type:

  TABLE

- Tell LogoSheet to draw a chart of the scores:

  SETAUTO "ON
  LHS *left-hand side*
  CC "Score "Frequency

- Now tally the results from the RED field and draw a chart on the right-hand side of the screen.

  TALLY "RED "Score "Freq
  RHS
  CC "Score "Freq

Can you judge which is the unfair dice? Did the distributions in the two charts look sufficiently different?

. . . This dice is unfair!

Clue 4: Runs

Perhaps the dice is unfair and produces runs of sixes, such as five sixes.

- A scatter graph may help you.
  For example:

FS *full screen*
CS
SG "ROLL "BLUE

Can you see how to spot any runs from this graph?

The problem is that a fair dice is bound to produce some runs.

Do either of the dice have too many (or too few) runs?

Are any of the runs unusually long?

- Now can you decide which is the unfair dice?

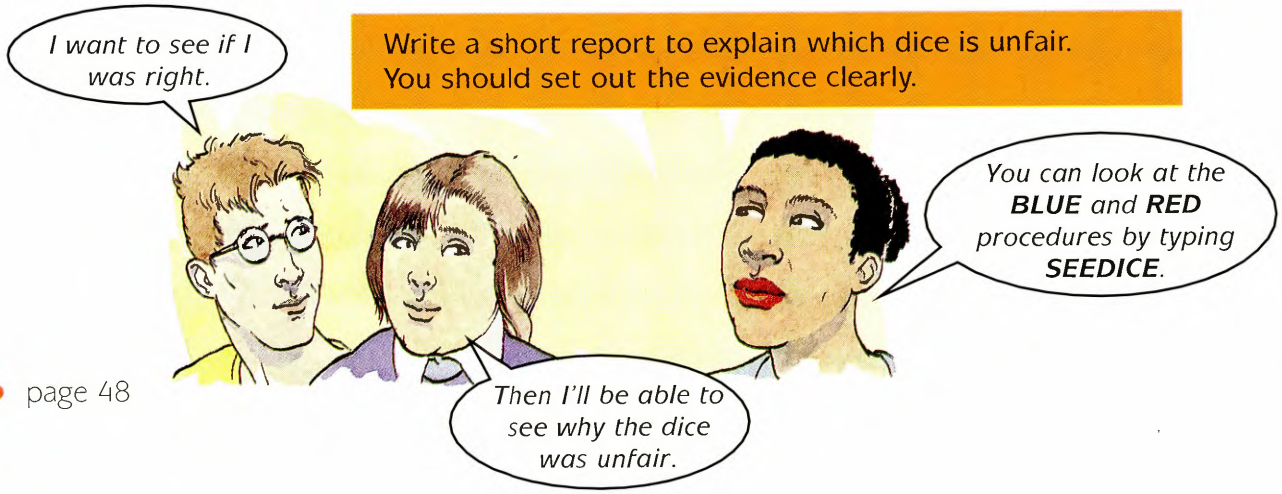
However, it is reasonable to feel that you need more data.

You can add 20 more rolls of each dice by typing:

REPEAT 20 [ROLL]

Now you may want to look again at some of the clues.

Write a short report to explain which dice is unfair.
You should set out the evidence clearly.



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The colour machine

Some more ideas

Here are some more ideas. You do not need to use them all but one or two might interest you. Of course you might have some better ideas ...

I want to use fruit instead of colours.

Ok, but let's have more than three fruit. We could put three lemons and ... and ... into column A.

*We'll have to make the spreadsheet longer and we'll need to change the **WINS** and **PRIZES** procedures.*

I'd like to try out some different winning lines. How about fruit salad? In other words all three fruits have to be different to win.

I'd like to change the stake.

I'm going to see if I can have a different set of colours for each column. This will mean a re-write of the spreadsheet.

*Yes, **CHOOSE** uses the colours in cell A3 and below. We'll need a new procedure for cell B2 and a third one for C3.*

These new procedures will be like **CHOOSE** but they'll use the colours in columns B and C instead.

LogoSheet values

	A	B	C	D	E
1	RED	BLUE	GREEN		
2					
3	GREEN	GREEN	GREEN	STAKES =	0
4	BLUE	BLUE	BLUE	PAYOUT =	0
5	RED	RED	RED	PROFIT =	0
6	GREEN	BLUE	RED		
7	GREEN	BLUE	RED		
8	PINK	PINK	PINK		

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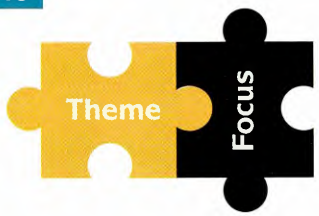
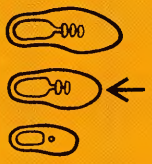


Here are some kinds of maths you may have used in this Theme.

You can find out more about them in these Focus book units.

Averages



Handling data
LI 1
C 1
E 7

Data collection and analysis

Handling data
C 8
E 8

Frequency tables and charts

Handling data
LI 5
C 11

Percentages

%

Number
LI 3
C 6

Probability

Handling data
LI 11
C 6
E 9

Ratio

1:5

Number
LI 3
C 6

Statistics

Handling data
LI 4
C 6
E 1, 2, 3

Using and interrogating data

Handling data
C 8
E 8

Theme Books

- provide open-ended starting points for mathematical activities in context;
- promote the natural development of cross-curricular work in a variety of subject areas.

Focus Books

- develop techniques and ideas in the four topic areas of NUMBER, ALGEBRA, SHAPE AND SPACE, HANDLING DATA;
- continue the investigative approach of the Theme Books, and are designed to be used alongside them.

Computer Activities are integral to the **Century Maths** materials, with fully referenced support through the **Logo 2000** software and **LogoPacks.**

This is the **ORDER TO... / ...RANDOMNESS Theme Book** for Years 10 and 11.

ORDER TO...
...RANDOMNESS

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